NONTRIVIAL STRUCTURE OF SYNCHRONIZATION ZONES IN MULTIDIMENSIONAL SYSTEMS

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The structure of the synchronization regions in a system with many degrees of freedom is investigated experimentally on a microwave oscillator with a ferrite resonator.

The structure of the synchronization regions emerging in the coupling of double-frequency oscillations in relatively simple systems has been studied fairly well. Models with a small number of parameters, for example, one-dimensional maps of a circle and two-dimensional maps of a ring into itself [1–3] can be used in this case. However, synchronization types and regions may be more diverse and qualitatively different in systems with many degrees of freedom, for example, microwave oscillators [4,5]. In spite of plenty of experimental data there are, in fact, no papers concerned with a systematic analysis of synchronization phenomena in systems with a multidimensional phase space. Our paper deals with the investigation of the synchronization regions typical of multimode systems and differing essentially from those studied earlier.

The synchronization in a driven microwave self-oscillator with a ferrite resonator was investigated experimentally. It was established that the transition to chaos in such an oscillator is related to the resonant interaction of three resonator modes having close frequencies [5–7]. Therefore the dynamics of such a system is described approximately by a set of six differential equations that depend on three parameters [5], which is indicative of a more complex, as compared to ref. [12], bifurcation structure of synchronization.

The parameters of the self-oscillator in the experiment were chosen such as to provide the transition to chaos via period doubling in the absence of external modulation. The value of the magnetic field $H$ determining the frequency of the resonator eigenmodes, the amplitude $A$ and the frequency $\omega$ of external modulation were the controlled parameters. The projections of the phase space, the Poincaré maps through the external signal period and the bifurcation $H$-diagrams were registered.

Fig. 1 shows the partitioning of the parameter space in the $(A, H)$ plane for $\omega=\text{const}$ and the $(A, \omega)$ plane for $H=\text{const}$ in the vicinity of the synchronization point $1:2$ between the self-modulation and the external signal frequencies. The index $+1$ denotes the bifurcation curves bounding the “horn” of the resonance of interest. The index $-1$ is for the bifurcation curves for the period doubling of the motions inside the synchronization horn and along the $A=0$ axis and for the two-dimensional ergodic tori outside the resonance regions. The boundary of the domain of smooth ergodic tori is designated through $\gamma$. (One of the torus multipliers turns to zero on the $\gamma$ curve.)

$^1$ Here we mean a self-modulation period. The high-frequency carrier background may be neglected because its period is more than 1000 times shorter than the characteristic time of measuring self-modulation and external effect. Mathematically this means the study of the phenomena that occur in a system averaged over high frequency.

$^2$ The construction of the bifurcation diagram was described in refs. [5,8].

$^3$ Similar structures of synchronization regions were also observed at other resonance points.
Below the curve $\gamma$ a stable and a saddle cycle are born on a two-dimensional torus in the transition across the bifurcation curve +1 to the resonance region. When $\lambda$ is small, a fine structure of higher-order resonances (which is omitted in fig. 1 for simplicity) is formed in the neighborhood of the main resonance. A fragment of the bifurcation diagram of such a structure along the curve $S_1$ is shown in fig. 2a. When the curve $\gamma$ is crossed with increasing parameter $\lambda$ along the curve $S_2$ the torus is corrugated (fig. 3e) and then destroyed with the formation of a strange attractor $SA_1$ (fig. 3h).

A cascade of period doubling bifurcations of motion and the birth of a strange attractor $SA_2$ are recorded inside the synchronization horn (figs. 3b–3d). The corresponding bifurcation diagram along the curve $S_3$ is presented in fig. 2b. The intersection of the curve −1 outside the synchronization horn (fig. 1a) corresponds to the doubling of a two-dimensional ergodic torus. Inside the horn there occur doublings of the synchronized limit cycles together with the invariant torus on which they are located. In this case, a saddle cycle is doubled simultaneously with a stable limit cycle due to the presence of an invariant torus.

The situation is more complicated when the ex-
Fig. 3. Maps through external drive period for parameter values denoted by the corresponding letters in fig. 1b.
ternal drive has a large amplitude $A$ (i.e., above the curve $\gamma$). Then the doubling occurs with nonsmooth (corrugated) invariant tori. The intersection of the curve $\pm 1$ outside the horn above the curve $\gamma$ (fig. 1b) corresponds to the merging of stable and saddle cycles with the formation of a stable corrugated torus. In this case, the doubling curves $\pm 1$ may be extended beyond the horn and correspond to the doubling of such corrugated tori. This follows from comparison of figs. 3e and 3g. The intersection of the curve $\pm 1$ above the critical doubling curve $\xi_c$ causes the crisis of a Feigenbaum-type strange attractor $SA_2$ (fig. 3d) and the birth of a new attractor $SA_1$ (fig. 3h) that corresponds to a more complicated chaotic motion of higher dimension.

The bifurcations considered above cannot be described within the model presented in ref. [2] (cf. figs. 1a–1c) because the doubling and the merging of stable and saddle limit cycles on tori proceeds in different directions in the phase space (for mapping). Therefore they can be used only with three- (or higher)-dimensional maps (or with a system having continuous time and a dimension not lower than four). Experimental observation of such a structure indicates that a mathematical model must have a rather high dimension.

Note in conclusion that the bifurcation phenomena at the boundaries of synchronization zones of multidimensional systems are rather diverse. In particular, the investigation of the resonance 1:1 in a certain parameter region has shown that a stable and a saddle periodic orbit with complex-conjugated multipliers merge when leaving the synchronization zone. The evolution of “spiral tori” with decreasing distance from resonance typical of this case is shown in fig. 4 and the bifurcation diagram for the parameter $H$ is presented in fig. 5.

References