Thin-film magnetic patterns in an external field

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The behavior of ideally soft ferromagnetic films in the presence of a weak coplanar magnetic field is explored by the method of characteristics. Solutions are found which have no internal field or in some cases have field-free zones. Results are specifically given for circular and elliptic disks, the infinite strip, and the semi-infinite plane. The disk solutions have internal domain walls.

In this letter we consider the magnetization patterns which occur in a thin ferromagnetic film of soft material (anisotropy neglected) subjected to a weak coplanar magnetic field. The analysis is carried out explicitly for the simple cases of an infinite strip, and semi-infinite plane, as well as for the more interesting cases of circular and elliptic disks. We assume that the domain wall thickness is small compared to the in-plane dimensions of the film, and that there is negligible variation in magnetization across the thickness Δ_z of the film, which will be ensured if the domain wall thickness is of the order of Δ_z or larger. In a sufficiently weak field, the sample may achieve a complete exclusion of the external field from its interior. In this case, we may compare the problem with the corresponding problem of a thin-film conductor in an electric field. There is, of course, a unique solution $\sigma(x,y)$ for the surface charge distribution on such a conductor. If a magnetization pattern can be found which exhibits this distribution of magnetic charge, then such a pattern will be stationary (since field is excluded) and possesses the minimum possible energy. The charge is actually distributed on the upper and lower surfaces, but for a sufficiently thin sample it may be approximated as permeating the film, giving rise to a pseudodivergence of M $[\nabla\cdot M \equiv \partial M_x/\partial x + \partial M_y/\partial y = -\sigma/\Delta_z]$. This is in contrast to the case of cylindrical objects[1] where a cross section has charge only along the perimeter. Applied and demagnetizing fields are assumed weak—of order $M_s\Delta_z/L$, where L is a typical length across the sample (or field-free region of the sample) and M_s is the saturation magnetization. As a result, demagnetizing effects force the magnetization to remain almost entirely confined to the plane and also parallel to the edge along the perimeter of the sample. Any deviation from this behavior would produce strong fields (of order M_s) tending to oppose this deviation.

If the magnetization is specified by its angle θ relative to the x axis ($|M| = M_s$ is assumed constant), then defining $m \equiv M/M_s$ we obtain

$$\nabla\cdot\mathbf{m} = -\sin\theta\frac{\partial\theta}{\partial x} + \cos\theta\frac{\partial\theta}{\partial y} = -\frac{\sigma(x,y)}{\Delta_z M_s}. \quad (1)$$

This may be solved by the method of characteristics[2] where the characteristic curves are given by

$$\frac{dx}{ds} = -\sin\theta, \quad \frac{dy}{ds} = \cos\theta, \quad \frac{d\theta}{ds} = -\frac{\sigma(x,y)z}{\Delta_z M_s}. \quad (2)$$

The first two equations show that the characteristics are always perpendicular to $\mathbf{M}$. The third equation shows that the curvature of the characteristics is proportional to the local charge density. This is in contrast with the case of no external field (see van den Berg[3]) in which characteristics are straight lines. The characteristic curves originate normal to the boundary and curve as they move inward. Within the sample these must be connected together at domain walls. These walls cannot be put in arbitrarily; it is necessary that the angle of incidence between the characteristics and the wall be equal on both sides, to avoid deposition of magnetic charge on the wall.

For our first example, we consider a thin-film strip of magnetic material extending from $-a$ to $+a$ in the x direction from $-\infty$ to $+\infty$ in the y direction and thickness Δ_z. This is in a uniform applied field H_0 across the width of the strip. For $\Delta_z \ll a$, the surface charge distribution which gives no internal field is

$$\sigma = H_0 x/2\pi\sqrt{a^2 - x^2}. \quad (3)$$

(One way to obtain this result is from an appropriate solution of Laplace's equation in elliptic cylindrical coordinates, see Jeans[4] for discussion of similar problems.)

Assuming no variation in y, we may immediately integrate Eq. (1), obtaining

$$m_x = (H_0/2\pi\Delta_z M_s)\sqrt{a^2 - x^2}. \quad (4)$$

This solution [shown in Fig. 1(a)] is valid only as long as $H_0 < 2\pi M_s\Delta_z/a$ since we require that $|\mathbf{m}| = 1$. For stronger fields field penetration must occur. This fact is borne out by a numerical solution, the results of which are plotted in Figs. 1(b) and 1(c). As can be seen, the field penetration is limited to a central zone. There remain two boundary zones which still have zero field.

Another case that is simple to analyze is the semi-infinite plane. Here one finds that for any nonzero applied field H_0, there is always a field-free boundary zone (FFBZ) of finite width a, beyond which there are field penetration and uniform magnetization.

Let us first assume the existence of such a solution (we will show it is stable later). Since magnetization is uniform outside of the FFBZ, magnetic charge exists only inside the FFBZ and we can once again solve for this by looking at the correct electrostatics problem. An important difference between the FFBZ and the strip considered previously is that the total charge inside of the FFBZ is not zero: since M_x changes from 0 to 1 when crossing the zone we must require that

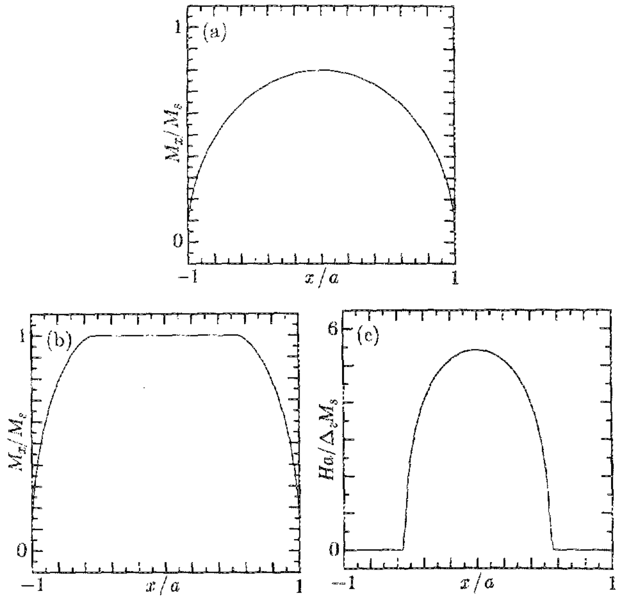

FIG. 1. Magnetization and field in an infinite strip. (a) Plots x component of magnetization as function of x for $H_0 a/\Delta_z M_s = 5$. This case has $H = 0$ inside the strip. (b) Here $H_0 a/\Delta_z M_s$ is increased to 10. In the central zone $M_x = M_s$ and field penetration occurs as shown in (c), a plot of the internal field. Note there remain field-free zones near the boundaries.

$$\int_0^a \sigma\, dx = -\Delta_z M_s. \tag{5}$$

Thus this case corresponds to a charged conductor in an electric field. For the total charge given above, and external field H_0, we find

$$\sigma(x) = -\frac{4\Delta_z M_s + H_0(a/2 - x)}{2\pi\sqrt{x(a-x)}}. \tag{6}$$

We have yet to determine the appropriate value for the width of the FFBZ. If a is sufficiently large, then $\sigma(x)$ will increase to $+\infty$ as x approaches a. However, it is not possible for σ to be positive here because this would require that m_x be greater than unity for values of x just before $x = a$. If a is sufficiently small, on the other hand, $\sigma(x)$ will decrease to $-\infty$ as x approaches a. One can easily show that in this case the field for x slightly greater than a will be large and negative. This condition is unstable since the field is directed opposite to the magnetization. Clearly then, the correct choice for a is the one which is the boundary between these two cases, for which the charge goes to zero at $x = a$. Thus we find

$$a = 4\Delta_z M_s / H_0 \tag{7}$$

and σ can be expressed as

$$\sigma(x) = -H_0(a-x)/2\pi\sqrt{x(a-x)} \tag{8}$$

This is integrated to obtain the magnetization

$$\begin{aligned} m_x &= \frac{-2}{a\pi}\sqrt{x(a-x)} \\ &\quad - \left[\frac{1}{2} + \frac{2}{a\pi}\arcsin\left(\frac{2x-a}{a}\right)\right] \quad \text{for } 0 < x < a \\ &= 1 \quad \text{for } x > a. \end{aligned} \tag{9}$$

The field may also be obtained by integration

$$\begin{aligned} H(x) &= H_0 - \int_0^a \frac{2\sigma(x')}{x - x'}\, dx' \\ &= H_0\sqrt{1 - a/x} \qquad \text{for } x > a \text{ or } x < 0 \\ &= 0 \qquad \text{for } 0 < x < a. \end{aligned}$$

Note that the field is continuous at $x = a$, but discontinuous at $x = 0$, going to ∞ on approach from the negative side. Stability of the solution is assured by the fact that H is aligned with the magnetization in the penetration zone.

We now proceed to the case of the circular or elliptic disk, in a uniform applied field H_0, which requires numerical integration of the characteristic equation. The charge distribution in this case is found to be

$$\frac{\sigma}{\Delta_z M_s} = \frac{(4/\pi)(E_x x/a^2 + E_y y/b^2)}{\sqrt{1 - (x/a)^2 - (y/b)^2}},$$

where the x and y components of the reduced applied field are

$$E_x = H_{0x}\left(\frac{a^2 - b^2}{8M_s b\Delta_z(\tilde{K} - \tilde{E})}\right)$$

and

$$E_y = H_{0y}\left(\frac{b(a^2 - b^2)}{8M_s\Delta_z(a^2\tilde{E} - b^2\tilde{K})}\right),$$

where $\tilde{K}$ and $\tilde{E}$ are complete elliptic integrals of the first and second kind, respectively, of argument $\sqrt{1 - b^2/a^2}$, and where a and b are the semimajor and semiminor axes of the ellipse which are aligned with the x and y axes, respectively. (These expressions can be obtained from a solution of Laplace's equation in ellipsoidal coordinates; see Jeans[4] for a discussion of similar problems.)

For the case of a circular disk of radius a with field in the y direction these results reduce to

$$\sigma/\Delta_z M_x = 4yE/\pi\sqrt{a^2 - x^2 - y^2} \tag{10}$$

where the reduced applied field is

$$E = E_y = H_0 a/2\pi M_s \Delta_z. \tag{11}$$

We use this result to numerically integrate the characteristic equations. This gives the characteristic lines which are perpendicular to the magnetization. It is of course preferable to draw the m lines, i.e., lines parallel to the magnetization. This can be done by integrating characteristic curves from the boundary to the locus of the m line being followed. One can numerically evaluate the differential change in the endpoint of these curves with respect to the starting point on the boundary and use this information in a set of equations which will track the m lines. It is also possible to follow the domain wall (or walls) given a starting point on the wall and using the fact that the wall must bisect the angle between the characteristic curves reaching it from opposite sides. Both of these procedures were carried our for the circular disk, with results given in Fig. 2. For the circular disk it was found that a single c-shaped domain wall is required in all cases. One can show, by application of Gauss's law to the upper half of the disk [see Eq. (1)], that the wall must intersect the x axis at $-aE$. At $E = 1$ the wall reaches the disk boundary [Fig. 2(f)] and thus for higher fields the solution breaks down and we must assume that field penetration will occur.

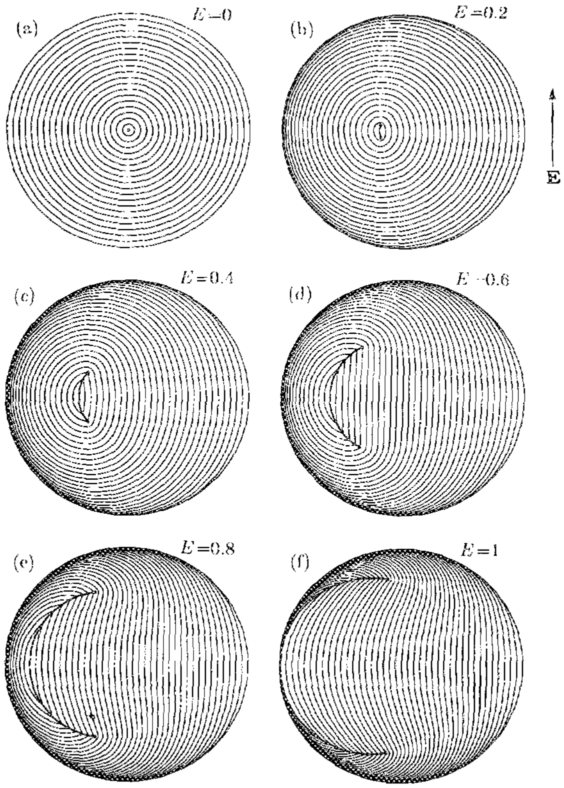

FIG. 2. m lines and domain wall (heavy line) for the circular disk with reduced applied field as indicated, $E = H_0 a/2\pi\Delta_z M_s$.

For a general boundary, location of domain walls becomes more difficult. Here we given a general method which should work in all cases for which the following assumption is valid. We assume that the required charge density is known and that every point in the interior has its magnetization direction determined by a characteristic line passing through that point which originates somewhere on the boundary. For a particular point A, we start by considering the family of characteristic curves passing through that point, as shown in Fig. 3(a). The correct curve must satisfy the boundary condition of normal incidence. If none of the curves intersect the boundary perpendicularly then the procedure fails (this may mean field penetration has occurred).

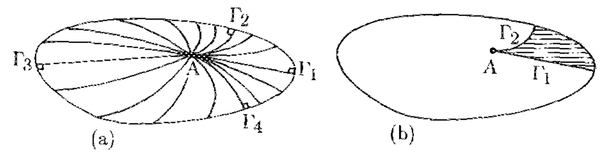

FIG. 3. (a) Family of characteristics for the point A. Four of these intersect the boundary at normal incidence, labeled Γ_1 through Γ_4. (b) Region of charge integration needed to compare the effective lengths of Γ_1 and Γ_4.

If one curve intersects perpendicularly, then this is the sought after characteristic for the point A. If two or more intersect perpendicularly, then the curve to be used is the one with the lowest (or most negative) effective length $\mathscr{L}_n$ defined as

$$\mathscr{L}_n = S_n - S_1 + \int_{\Gamma_1}^{\Gamma_n} \frac{\sigma}{\Delta_z M_s} dA,$$

where S_n is the length of the curve Γ_n and the integral is over the shaded area in Fig. 3(b) extending counterclockwise from the curve Γ_1 (assuming M goes that way also). The choice of the reference curve Γ_1 is arbitrary. Note that $\mathscr{L}_1 = 0$ and that negative $\mathscr{L}$'s are allowed. If the two lowest $\mathscr{L}$ values are equal, then point A is on a domain wall. If more than two are equal, then A is an intersection point between walls. The necessity for the $\mathscr{L}$ values to be equal on the wall follows from applications of Gauss' law [see Eq. (1)].

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