Noise Rise in Nondegenerate Parametric Amplifiers

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The noise-rise phenomenon is a long-standing problem that was first observed in Josephson-junction-parametric amplifiers over ten years ago. We present here a case where noise-rise data from a Josephson junction is successfully explained, using a theory based on the universal properties of a dynamical system operated near a bifurcation point. The experiment and theory presented here are for a bifurcation of the Hopf type, a case not discussed previously. The predicted behavior is qualitatively different from that of previously studied bifurcations.

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The ubiquitous noise rise in Josephson-parametric amplifiers was first observed in the late 1970's.[1-3] Early experiments with these devices were disappointing because it was found that the noise gain increased more rapidly than the signal gain as the operating parameters were adjusted. A number of explanations of this phenomenon came forth,[4-9] attributing the rise to such phenomena as phase instability, hopping between coexisting attractors, and deterministic chaos. Recently, the work of Bryant, Wiesenfeld, and McNamara[10,11] (BWM) showed that the effect could be accounted for as a universal feature of systems operated near certain types of instabilities or bifurcations. However, BWM did not generate any new experimental data in order to directly test the predictions of their theory. Bocko and Battiato[12] observed similar effects in a circuit employing a varactor diode, demonstrating the universal nature of this effect. However, our paper demonstrates for the first time that data from a Josephson-parametric amplifier can be successfully explained using the dynamical systems approach.

The BWM theory applies to systems operated near period doubling, symmetry breaking, and degenerate saddle-node bifurcations. Conspicuously missing from this list is the case of a secondary Hopf bifurcation, in which a second frequency (incommensurate with the first) begins to emerge at a certain operating parameter. Since the instability frequency in this case is not related to the pump frequency, we refer to this as the nondegenerate mode of operation. In this paper we present both theory and experimental data for this case, which, we will show, has significant differences from those studied previously. We now find that the noise rise is limited to a manageable 3 dB under certain operating conditions. Our experimental results for the nondegenerate Josephson-parametric amplifier display an excellent gain and low-noise behavior and good agreement with the theoretical results. The high performance of this amplifier provided strong motivation to make this detailed analysis of its behavior.

We begin our analysis by outlining the derivation of the reduced equation for this case. The reader will find more details on the general method of obtaining such equations from the earlier work of BWM.[11] We assume that the system is very near a Hopf bifurcation of a periodic orbit. In this case the dynamics can be approximated as lying entirely in the center manifold of the bifurcation, and in first approximation can be expressed as

$$\Phi(t)=\Phi_0(t)+[ze^{i\omega t}\Phi_1(t)+\text{c.c.}]\,. \qquad (1)$$

Here $\Phi_0(t)$ is the primary orbit of fundamental frequency ω_p (the pump frequency), $\Phi_1(t)$ is a complex vector function of fundamental frequency ω_p characterizing the instability, ω is the second frequency that emerges in the Hopf bifurcation, and z is a small and slowly varying complex amplitude factor. The exact forms of Φ_0 and Φ_1 depend on the system and on the choice of variables. Our experimental system is highly resonant near ω and $\omega_p-\omega$. As a result, Φ_0 is small since the system is off resonance at ω_p, and $\Phi_1\sim 1+e^{i\omega_p t}$. The behavior of z which we now describe is universal and applies to all similar systems. Under appropriate rescalings, z can be expected to satisfy the normal form equation[13]

$$\dot{z}=\mu z-z|z|^2+O(z^5)\,. \qquad (2)$$

In this equation μ is a parameter which crosses zero at the bifurcation point. Over some small interval near the bifurcation point it may be expected to vary approximately linearly with any actual parameter of a physical system. When crossing $\mu=0$, the fixed-point solution to the equation changes from $z=0$ to $|z|=\sqrt{\mu}$, corresponding to the emergence of oscillations of frequency ω. The dynamics in this case corresponds to a highly damped particle in a "Mexican hat potential."

Following an approach similar to BWM (Ref. 11) to

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include the effects of a single-frequency signal plus noise we can obtain an equation of the form

$$\dot{z} = \mu z - z|z|^2 + \epsilon e^{i\delta t} + \xi(t) , \quad (3)$$

where ϵ is proportional to the amplitude of the applied signal, δ is proportional to the detuning, i.e., the difference between the signal frequency and ω, and $\xi(t)$ is a complex white-noise term of unit strength, i.e., $\langle \xi(t)\xi^*(t+\tau)\rangle = \delta(\tau)$. Both z and t have been rescaled in obtaining Eq. (3) (the reduced equation) in order to set the coefficients of two of the terms on the right-hand side to unity. We have studied the dynamics of this equation analytically, and by digital simulations.

In the case of a noise-free system we drop the final term in Eq. (3). The presence of the detuned signal tends to phase lock the system at the signal frequency, inhibiting the emergence of the Hopf frequency. Transforming to coordinates rotating at the signal frequency ($z \to z e^{i\delta t}$) the system remains phase locked so long as there exists a stable fixed point. If the signal is weak, phase locking is lost for small positive μ via Hopf bifurcation. But for a sufficiently large signal the bifurcation switches to a saddle-node bifurcation, after which the dynamics follows an orbit near the minimum of the Mexican hat potential at $|z| = \sqrt{\mu}$. In the weak case the new peak will emerge from zero amplitude at a frequency distinct from the signal frequency. In the strong case the new peak will split away from the signal peak at finite amplitude. It can be shown that the signal power gain reaches a maximum of $1/\delta^2$ when $\mu = (\epsilon/\delta)^2$, a point just below where phase locking is lost.

We now consider the case of a system perturbed by both signal and noise near frequency ω. In general, there may be additional contributions to the noise gain generated by crossover from noise input at the frequencies $\omega_n = |n\omega_p - \omega|$, increasing the results we give below for G_n by a nearly constant factor C. For our experimental results, we observe $C \approx 2$ due to equal crossover from ω_1 (the other frequencies are off resonance and make negligible contribution). In the case of μ large and negative, we obtain equal noise and signal power gains of $G_s = G_n = 1/(\mu^2 + \delta^2)$. As μ is increased, we eventually reach the point where phase locking is lost as we described previously. Beyond this point, a new peak emerges on the opposite side of the Hopf peak (equally displaced in frequency), and grows to be of equal magnitude to the signal peak. A dynamically generated peak with this behavior is often called an idler. This peak may be difficult to see in a noisy system as it will be broadened to twice the width of the Hopf peak, with a corresponding decrease in amplitude. When μ is large and positive, the system dynamics will not vary far from the circle $|z| = \sqrt{\mu}$. A linearized analysis yields a signal power gain of $G_s = 1/4\delta^2$, which is a factor of 4, or about 6 dB, less than the maximum (noise-free) gain. The presence of the idler peak at $\omega - \delta$ allows input noise at that frequency to contribute equally to the response noise at the signal frequency. As a result, the noise power gain for large μ is $G_n = 1/2\delta^2$, a factor of 2, or about 3 dB, higher than the signal gain.

For large ϵ strong phase-locking effects occur. For μ relatively large but less than the point of maximum signal gain (ϵ^2/δ^2) we obtain the approximate results $G_n = \mu/2(\epsilon^2 - \delta^2\mu)$ and $G_s = \mu/\epsilon^2$. From these expressions we obtain two surprising effects: (1) For appropriate values of μ the noise gain can be less than the signal gain resulting in a negative noise rise of up to -3 dB, and (2) the noise gain goes to infinity near the point of maximum signal gain. Both results are nonlinear effects related to the large signal input.

We have also examined the effect of changing the input noise strength. By rescaling the variables needed to obtain Eq. (3) in the limit of weak signal and small detuning, we find that the maximum signal gain should be inversely proportional to the input noise power.

Numerical simulations were carried out both to verify the theory and also to examine the behavior in the nonasymptotic regimes where exact solutions are not possible. In Fig. 1 we show typical behavior of the noise rise (G_n/G_s) as a function of signal gain. In 1(a) we show a case of minimal phase locking; here $\delta = 1$ and $\epsilon = 0.3$. Note the 3-dB noise rise and the slight falloff in G_s. In 1(b) is shown a case of strong phase locking; here $\delta = 1$ and $\epsilon = 3$. This shows many features described by

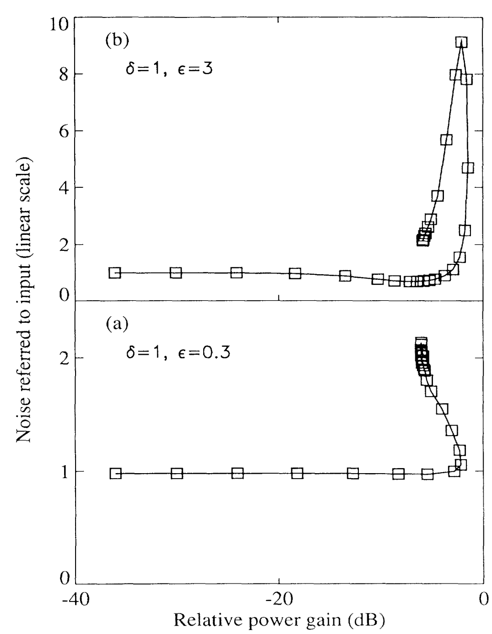

FIG. 1. (Noise gain)/(signal gain) (noise rise) plotted as a function of signal gain. δ and ϵ are kept fixed while μ is varied to make the plot. (a) $\delta = 1$ and $\epsilon = 0.3$. This is typical behavior when signal is weak. (b) $\delta = 1$ and $\epsilon = 3$. Strong phase-locking effects are evident.

the theory: a slight dip in the noise rise (here about −2 dB) followed by a sharp increase to 9.3 dB, followed by a slight decrease in G_s, and settling of the noise rise to a final value of about 3 dB.

We have conducted a systematic study of the noise performance of a Josephson-parametric amplifier as a function of pump power and input noise. The amplifier was pumped at near 38.5 GHz. Maximum gain occurs at two modes with frequencies symetrically spaced 200 MHz on either side of half the pump frequency. At the Hopf instability these modes break into oscillation. We will refer to the higher-frequency mode as the signal mode and the lower-frequency mode as the idler mode. The amplifier was previously used to demonstrate thermal noise squeezing[14] and quantum noise squeezing[15] and can exhibit extremely low-noise performance. When operated at 0.25 K the amplifier has exhibited gains in excess of 5000 with all amplifier noise accounted for by the input noise and the equilibrium fluctuations of the amplifier losses.

The instrumentation used is similar to that of previous experiments[15] except heterodyne detection rather than homodyne detection is employed. Noise of a known noise temperature along with a sinusoidal signal were injected into the input port of the parametric amplifier. The injected signal was tuned to have a frequency close to that of the signal mode frequency. Since we were measuring the large-gain behavior of the amplifier, the amplifier's gain and output noise floor could be measured directly from the spectrum analyzer without signal averaging. A waveguide switch, which allowed one to replace the amplifier with a section of waveguide, made possible accurate gain and loss measurements. Noise powers were calibrated using a variable-temperature cold termination located at the input port of the parametric amplifier.

Figure 2 shows the noise referred to input (output noise divided by the signal power gain) plotted against the signal power gain for differing amounts of input noise power. Care was taken to make the signal power sufficiently small that its presence did not have a measurable effect on the noise power spectrum. That is, care was taken to insure that one was in the weak-signal case corresponding to that of Fig. 1(a). The measured amplifier losses for these data sets are 2.78 dB for the 81-K curve, 2.09 dB for the 26-K curve, 2.55 dB for the 7.7-K curve, 3.27 dB for the 2.4-K curve, and 3.64 dB for the 0.48-K curve. From the temperature of the amplifier (0.25 K for this run) and the measured losses one can calculate the expected small-pump-power noise floors, the dashed lines of Fig. 2. Note the good agreement between the expected and measured noise below the Hopf instability threshold. The equilibrium noise emitted by the losses and the noise entering the input of the amplifier thus fully account for the noise seen at the output of the amplifier for small pump power. As the pump power is increased the gain eventually saturates and the noise rises by 3 dB, as expected from theory. For the experiments of Fig. 2 a constant detuning between the signal frequency and the signal mode center frequency was not maintained. (The signal mode center frequency shifts by 10 MHz as the pump power is increased due to a change in the effective inductance of the Josephson junction.) Because of this the gain drops by more than the 6 dB expected from theory. From the figure one also sees that the maximum signal gain varies

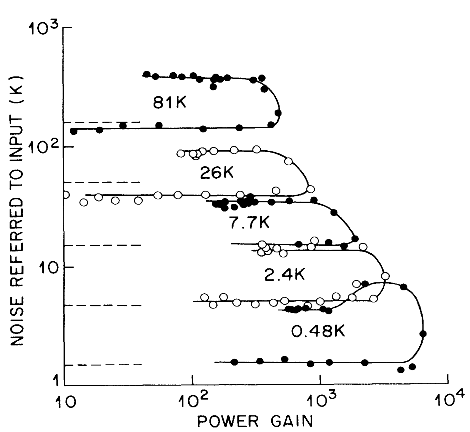

FIG. 2. Nosie referred to input as a function of signal power gain for varying amounts of input noise. The expected below-threshold values are indicated by dashed lines.

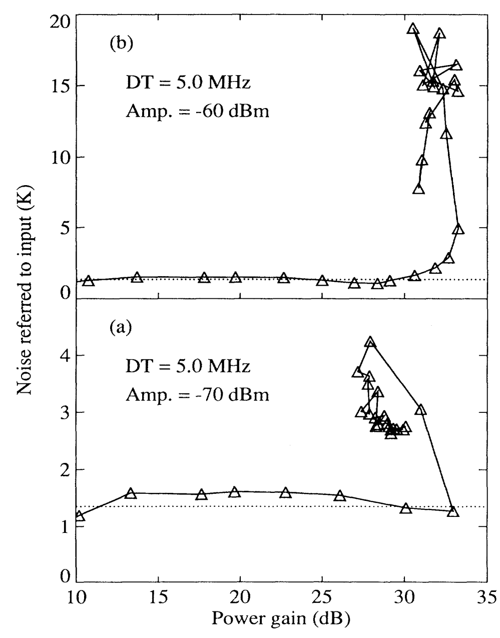

FIG. 3. Noise referred to input as a function of signal power gain. The detuning parameter has been held fixed. All the parameters of (a) and (b) are identical except the input signal is 10 dB greater in (b).

roughly inversely with the input noise in agreement with the theory. For the runs of Fig. 2, the detector noise temperature was 2440 ± 240 K and the amplifier was pumped at 38.6 GHz. The mixer local oscillator frequency was 19.00 GHz. The vacuum noise floor $h\nu/2k$ is 0.46 K. The equilibrium noise at 0.25 K is only 5% larger than this. Hence the 0.48-K run is one where the system is largely driven by vacuum fluctuation noise.

The data of Fig. 3 were taken at constant detuning as follows. First with the signal turned off the signal mode center frequency was determined by locating the maximum of the noise power spectrum. The signal frequency was then detuned a fixed amount of 5 MHz from the measured signal mode center frequency. The signal was then turned on. For the runs of Fig. 3, the pump frequency was chosen to be 38.4 GHz, the local oscillator was chosen to be 20.00 GHz, and the amplifier was cooled to 50 mK. Cryogenic microwave amplifiers[15] were employed to decrease the detector system noise temperature to 330 K. The amplifier's loss was measured to be 3.3 dB. The expected noise referred to input for small pump power is 1.35 K. Shown as dotted lines this noise compares favorably with the measured value of 1.5 ± 0.2 K. The data of Figs. 3(a) and 3(b) were taken under identical conditions except the signal power was 10 dB larger for the run of Fig. 3(b). Note the qualitative comparison between Fig. 1(a) and Fig. 3(a). Figure 3(a) corresponds to the case when the signal is sufficiently weak to prevent significant phase locking of the signal mode. Note that again the noise saturates at 3 dB above the low-pump-power value when the pump is made large. The good qualitative comparison between Fig. 1(b) and Fig. 3(b) indicates that Fig. 3(b) corresponds to the case when the input signal is sufficiently strong to force the signal mode to phase lock with the input signal. Note also that, in agreement with theory, the gain does not drop by more than 6 dB below its maximum value as the pump power is increased.

We have studied parametric amplification in nondegenerate systems in both theory and experiment. The analysis has been based on the normal form for a Hopf bifurcation, which is the instability responsible for the high gain achieved in this operating mode. The experimental results from a Josephson-parametric amplifier are all in agreement with theory. The theory achieves quantitative agreement in predicting the asymptotic level of noise rise (3 dB), a drop of the high-pump-power gain of no more than (6 dB) below that of the maximum gain provided the detuning is held fixed, and the inverse relation between maximum signal gain and input noise level. The qualitative agreement between theory and experiment in the parameter region where the excess noise becomes large is also good. Work is in progress to measure the quantities necessary to make this comparison quantitative as well. The theory is universal and will apply to any parametric amplifier (not just Josephson-junction based) if operated in the nondegenerate mode.

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